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WAVE GUIDE PROPERTIES OF HETEROGENEOUS MEDIA

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The normal wave method furnishes a large-scale picture of wave phenomena originating beyond the limits of the horizon, when a heterogeneous troposphere and an ionosphere are present. In particular, it forecasts the existence of wave guide channels lying on the surface of the ground or raised above the ground.

This article is a summary of Chapters XI to XIV of the author's thesis, published in November 1945 and also published in part in his book entitled, "The Normal Wave Method as Applied to the Problem of Radio Telecommunications" [1].

The connection between the problem studied here and the problem of wave guides of complex form was established by the author in a previous work [2].

We recently found that this problem had been independently solved in England by Booker [3] and in the United States by Ferris, Frigafer and Pekeris [4].

I. PLANE-STRATIFIED MEDIUM

A. Statement of the Problem -- Normal Wave Method

Let us examine an isotropic, unlimited, nonmagnetic nonresisting medium. We shall further assume that the dielectric constant ε varies only along one coordinate z, which is the axis of the cylindrical system of coordinates r, θ, z. Electromagnetic waves are propagated by vertical currents of frequency ω, distributed with arbitrary current density I (r,z,), independent of the angle θ. The electromagnetic field can now be expressed by the scalar equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A}{\partial r}\right)+\frac{\varepsilon\omega^2}{c^2}\frac{\partial}{\partial z}\left(\frac{c^2}{\omega^2\varepsilon}\frac{\partial A}{\partial z}\right)+\frac{\varepsilon\omega^2}{c^2}A=-\frac{4\pi}{c}I, \qquad (1)$$

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where A is the z-axis component of the potential vector **A** which determines H_θ, E_r and E_z.

The solution of equation (1) can be put in the form of a spectrum with a discrete and continuous portion

$$A(r,z)=\sum_{j=1}^{\infty}\varphi_j(r)Z_j(z)+\int\varphi(r,\chi)Z(z,\chi)\,d\chi \qquad (2)$$

where Z(z) are proper functions, and χ the proper values of the differential equation

$$k^2\varepsilon\frac{d}{dz}\left(\frac{1}{k^2\varepsilon}\frac{dZ}{dz}\right)+k^2\varepsilon Z+\chi Z=0, \qquad (3)$$

and $\varphi(r)$ are the Fourier coefficients

$$\varphi(r)=\int A(r,z)\,Z(z)\,dz. \qquad (4)$$

Substituting (2) in (1) and taking orthogonal properties and normalizing conditions into consideration, we find that $\varphi(r)$ satisfies the equation:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\varphi_j}{dr}\right)-\chi_j\varphi_j=-\frac{4\pi}{c}\,\bar{I}_j(r), \qquad (5)$$

where

$$\bar{I}_j(r)=\int I(z,r)\,Z_j(z)\,dz \qquad (6)$$

We omit the analogous equation for the continuous portion of the spectrum.

Equation (5) can be solved by means of Green's function $K(r,\rho)$, which is subject to: (1) boundary emission conditions at r = 0; and (2) the boundary condition that it be finite when $r\to\infty$. We shall then obtain:

for

$$r>\rho \quad K_j(r,\rho)=-\frac{1}{d_j}J_0(\sqrt{-\chi_j}\rho)\,H_0^{(2)}(\sqrt{-\chi_j}r), \qquad (7)$$

for

$$r<\rho \quad K_j(r,\rho)=-\frac{1}{d_j}H_0^{(2)}(\sqrt{-\chi_j}\rho)\cdot J_0(\sqrt{-\chi_j}r),$$

where

$$d_j=r\left[J_0(\sqrt{-\chi_j}\rho)\,H_0^{(2)\prime}(\sqrt{-\chi_j}\rho)-J_0'(\sqrt{-\chi_j}\rho)\,H_0^{(2)}(\sqrt{-\chi_j}\rho)\right]=$$
$$=-\frac{2i}{\pi}, \qquad (8)$$

where J_0 and H_0 are Bessel and Hankel functions of the zero order.

As a result

$$\varphi_j(r)=\frac{\pi^2}{ic}\int_0^{\infty}\bar{K}_j(r,\rho)\rho\,\bar{I}_j(\rho)\,d\rho, \qquad (9)$$

and the initial quantity A(r,z), in accordance with (2), has the final form

$$A(r,z)=\frac{2\pi^2}{ic}\sum_{j=1}^{\infty}Z_j(z)\int_0^{\infty}\bar{K}_j(r,\rho)\rho\,\bar{I}_j(\rho)\,d\rho+\int\cdots d\chi. \qquad (10)$$

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where $\bar{K}_j(r, \rho)$ is equal to $d_j K_j(r, \rho)$, and the unbroken portion of the spectrum is expressed by the symbol $\int \ldots d\chi$.

If the sources are concentrated within a circle of radius R, then for $r > R$

$$A(r,z)=\sum_{j=1}^{\infty} C_j Z_j(z) H_0^{(2)}(\sqrt{-\chi_j}\, r)+\int \cdots d\chi. \qquad (11)$$

In view of the asymptotic expression for the Hankel functions when r is large, we may consider expression (11) as a normal wave spectrum analogous to a wave guide spectrum. These waves are propagated to infinity with various amplitudes

$$C_j=\frac{2\pi^2}{ic}\int^{R} J_0(\sqrt{-\chi_j}\,\rho)\,\rho\,\bar{I}_j(\rho)\,d\rho. \qquad (12)$$

Every normal wave is characterized by its own wave number $\sqrt{-\chi}$ and by the form of amplitude distribution along the front Z(z). Proper negative values propagate nondamping normal waves, but proper positive values are rapidly damping normal waves which can only be observed near the antennae.

If the vertical currents I(r,z) are equally distributed within a circle of radius R, the amplitudes of a normal wave will be determined by the following expressions:

$$C_j=\frac{2\pi}{ic} I_j \frac{J_1(\sqrt{-\chi_j}\,R)}{\sqrt{-\chi_j}\,R} \qquad (13)$$

where

$$I_j=\pi R^2 \bar{I}_j=\int I(z) Z_j(z)\,dz \qquad (13')$$

is the component of the full current I(z), flowing across the cross section of a circle of radius R, propagating normal wave number j.

In the case of an infinitely fine antenna, $C_j = \frac{\pi}{ic} I_j$.
where:

$$A(r,z)=\frac{\pi}{ic}\sum_{j=1}^{\infty} I_j Z_j(z) H_0^{(2)}(\sqrt{-\chi_j}\, r)+\int \cdots d\chi. \qquad (14)$$

If the antenna length converges toward zero, we can substitute for it an infinitely small dipole P = el, located at the point $z = \zeta$, r = 0 and in accordance with (14),

$$A(r,z)=\frac{\pi}{ic}\sum_{j=1}^{\infty} Z_j(\zeta) Z_j(z) H_0^{(2)}(\sqrt{-\chi_j}\, r)+\int \cdots d\chi. \qquad (15)$$

To excite a narrow ray in the medium, it is possible to utilize an antenna with the following current distribution:

$$I(z)=0 \text{ for } z>\Delta z \text{ and } z<-\Delta z;\ I(z)=e^{-in_0 z} \text{ for } \Delta z \geqslant z \geqslant -\Delta z. \qquad (16)$$

If $\varepsilon(z)$ is a slowly varying function of z and the spectrum χ can, therefore, be replaced by a continuous spectrum, such an antenna will excite a packet of normal waves with width:

$$\Delta\nu=\frac{4\pi}{l}\frac{2\nu}{\nu_0} \qquad (17)$$

where ν_0 is the wave number of the central wave of the packet:

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$$\nu_0 = \sqrt{k^2 \varepsilon(0) - \varkappa_0^2}. \tag{18}$$

Within the limits of approximation of geometrical optics, the result of the superposition of a packet of normal waves gives a narrow ray with a trajectory:

$$r = \int^{z} \left(\frac{d\sqrt{k^2 \varepsilon - \nu^2}}{d\nu} \right)_{\nu=\nu_0} dz = \nu_0 \int^{z} \frac{dz}{\sqrt{k^2 \varepsilon - \nu_0^2}} . \tag{19}$$

It is easy to show that

$$k\sqrt{\varepsilon} \sin\alpha = k\sqrt{\varepsilon(0)} \sin\alpha_0 = \nu_0 = const, \tag{20}$$

where α is the angle between the tangent to the trajectory and the axis oz.

B. <u>Ideal Wave Guide Channels</u>

The simplest case is a homogeneous medium where $\varepsilon = 1$, included between two absolutely conducting planes $z = 0$ and $z = h$. Equation (3) with the following boundary conditions

$$\frac{dZ}{dz} = 0 \text{ for } z = 0 \text{ and } z = h \tag{21}$$

gives the discrete spectrum of proper values

$$\chi = \frac{\pi^2 j^2}{h^2} - k^2, \; j = 0, 1, 2, 3 \ldots \infty. \tag{22}$$

and proper functions [eigenfunctions]:

$$Z_0 = \frac{1}{h}; \; Z_j = \sqrt{\frac{2}{h}} \cos \frac{\pi j}{h} z; \; j = 1, 2, 3 \ldots \infty \tag{23}$$

In the case of an infinitely small dispole, equation (15) takes the form:

$$A(r, z) = \frac{\pi P}{i c h} \left\{ H_0^{(2)}(kr) + 2 \sum_{j=1}^{\infty} \cos \frac{\pi j \zeta}{h} \cos \frac{\pi j z}{h} H_0^{(2)}\left(\sqrt{k^2 - \frac{\pi^2 j^2}{h^2}}\, r \right) \right\} \tag{24}$$

This expression coincides with Weirich's formula [5].

In the case of a directed antenna (16), we get a narrow ray with a zigzag trajectory."

Before examining the heterogeneous media, by substituting

$$Z(z) = \sqrt[4]{\varepsilon}\, \psi(z)$$

we convert equation (3), into a Schrödinger equation

$$\frac{d^2 \psi}{dz^2} + [\chi + W(z)]\, \psi = 0, \tag{25}$$

where

$$W(z) = k^2 \varepsilon - \sqrt{\varepsilon} \left(\frac{1}{\sqrt{\varepsilon}} \right)_{zz} \tag{26}$$

plays the role of a potential function.

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Let us now examine the case where the wave guide channel is formed in spite of the absence of the sharp limits which obtain in the Weirich instance.

Let $z = z_1$ and $z = z_2$ be two axial points of equation (25) in which $W(z)$ converges toward $K \to \infty$. If $W > 0$ in the restricted region z, as shown in Figure 1, we shall obtain the discrete spectrum χ, starting from negative values.

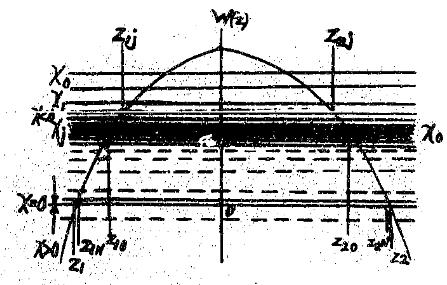

Figure 1

At great distances we can limit the series of normal waves (11)

$$A(r,z) \cong \frac{k\sqrt{\varepsilon}}{\sqrt{r}} \sum_{j=1}^{N} \frac{c_j}{\sqrt{|\chi_j|}} \psi_j(z) e^{-i\left(\sqrt{|\chi_j|}\,r - \frac{\pi}{4}\right)}, \qquad (27)$$

where χ_N is the smallest proper negative value.

Expression (27) demonstrates that a medium with a discrete spectrum with proper values χ has the typical properties of a wave guide channel because: (1) the amplitudes of normal waves are definite only between the reverse planes $z = z_{1N}$ and $z = z_{2N}$; and (2) the average value of the field intensity decreases as $\frac{1}{\sqrt{r}}$, since the second term (27) is a quasi-periodical function of r. Waves are propagated in a two-dimensional channel; therefore, undergo considerably less damping than in the case of wave propagation in free unlimited space.

If a directed antenna (16) be placed in the channel, a ray develops subject to liberation between the two reverse planes $z = z_{10}$ and $z = z_{20}$, as a result of successive reactions in the channel "walls."

An example of an ideal overground channel is furnished by the case produced by a potential function

$$W(z) = A - Bz^2.$$

Writing $\chi + A = \lambda$, we arrive at the well-known equation of wave mechanics for a harmonic oscillator

$$\frac{d^2\psi}{dz^2} + [\lambda - Bz^2]\psi = 0, \qquad (28)$$

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which, in the case of a dipole, gives the following expression for the field in the channel

$$A(r,z) = \frac{\pi P}{ic} e^{-\frac{z^2}{2}} e^{-\frac{\zeta^2}{2}} \sum_{j=1}^{\infty} H_j(\zeta) H_j(z) H_0^{(2)}(\sqrt{-\chi_j}\, r) \tag{29}$$

where H_j is a Hermitian polynomial and

$$\chi_j = -A + \sqrt{B}(2j+1) \quad j = 0, 1, 2, 3 \ldots \tag{30}$$

If the return planes are located at a great distance one from the other, the directed antenna (16) will generate a ray the trajectory of which is determined by (19)

$$z = \sqrt{\frac{\varepsilon_0}{\gamma}} \cos\alpha_0 \sin\left\{\sqrt{\frac{\gamma}{\varepsilon_0}} \frac{r}{\sin\alpha_0}\right\}, \tag{31}$$

where

$$\varepsilon = \varepsilon_0 - \gamma z^2 .$$

An example of an ideal surface wave is given by the potential function

$$W(z) = \varepsilon k^2 = \frac{w^2}{a^2 z^2}, \tag{32}$$

which corresponds to the linear increase of phase velocity c with height: $c = az$.

The solution of equation (25)

$$\frac{d^2\psi}{dz^2} + \left[\chi + \frac{\omega^2}{a^2 z^2}\right]\psi = 0 \tag{33}$$

is expressed by a Hankel function of the first order and

$$Z(z) = k\sqrt{\varepsilon}\,\psi(z) = A\frac{\omega}{a\sqrt{z}} H_{ip}^{(1)}(i\sqrt{|\chi|}\, z) \text{ for } \chi > 0, \tag{34}$$

where

$$p = \sqrt{\frac{\omega^2}{a^2} - \frac{1}{4}} .$$

The boundary conditions (21) on an ideal ground plane will select a discrete spectrum of proper values χ by means of the transcendental equations

$$\frac{d}{dz}\left(\frac{1}{\sqrt{z}} H_{ip}^{(1)}(i\sqrt{|\chi_j|}\, z)\right)\Big|_{z=z_0} = 0. \tag{35}$$

The dipole field located at the point $z = \zeta$ will be

$$A(r,z) = \frac{\pi P}{ic}\sqrt{2\zeta} \sum_{j=1}^{\infty} \frac{H_{ip}^{(1)}(i\sqrt{|\chi_j|}\,\zeta) H_{ip}^{(1)}(i\sqrt{|\chi_j|}\, z) H_0^{(2)}(\sqrt{-\chi_j}\, r)}{\left\{\int_{z_0}^{\infty} \frac{1}{z}[H_{ip}^{(1)}]^2 dz\right\}^{\frac{1}{2}}}. \tag{36}$$

If the directed antenna be taken as an oscillator, there will be produced in the medium a ray subject alternately to reaction in the heterogeneous medium and to reflection from the earth so that its trajectory will have the form of arches, expressed by formula:

$$z = \sqrt{\frac{\omega^2}{a^2 \chi_0^2} - r^2},$$

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where $\nu_0 = k\sqrt{\varepsilon(z_0)} \cdot \sin\alpha_0$ is the central wave of the packet.

C. Space Pulsations and Resonance Between Adjacent Channels

From the standpoint of the wave theory it is easy to understand how energy radiated by an antenna on the ground can be generated in an overground wave guide channel.

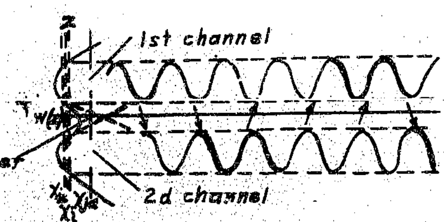

Figure 2

In the simple case of a medium with potential function W(z), represented on the left side of Figure 2, after augmenting the depression e in W(z), we shall intensify the potential barrier between the channels. One of the channels may be conditionally regarded as overground and the other as on the surface, although the function W(z) shown in Figure 2 more nearly corresponds to two active overground channels. Within an infinitely deep barrier we shall have two noninteracting channels with identical proper values χ_j ; and proper functions ψ_j . This is a case of degeneration. As a result of interaction through the potential barrier, each proper value of the degenerate case χ_j is split into two, $\chi_j + \Omega_j$ and $\chi_j - \Omega_j$, corresponding to a symmetrical and an antisymmetrical proper function. Applying the BVK method, we obtain

$$\chi_j(a,s) = \chi_j \pm \Omega_j \cong \chi_j \pm \frac{e^{-2\int_{z_2}^{z}\sqrt{k^2\varepsilon - \chi_j}\,dz}}{2\left[\frac{d}{d\chi}\int_{z_1}^{z_2}\sqrt{k^2\varepsilon - \chi}\,dz\right]_{\chi=\chi_j}}. \qquad (37)$$

Let us now examine an infinitely fine antenna with a current distribution $I(z) \cong \psi_j$ in the first channel and $I(z) = 0$ in the second channel, where $\psi_j(z)$ is one of the proper functions of the degenerate case. Now in each of the channels both normal waves with distributions ψ_{ja} and ψ_{js} will develop simultaneously. At great distances from the source, the whole field will have approximately the following form in the first channel:

$$\frac{k\sqrt{\varepsilon}}{\sqrt{r}}\psi_j(z)\cos\left\{\frac{\Omega_j}{\sqrt{|\chi_j|}}r\right\}e^{-i\sqrt{|\chi_j|}\,r}, \qquad (38)$$

and in the second

$$\frac{k\sqrt{\varepsilon}}{\sqrt{r}}\psi_j(z)\sin\left\{\frac{\Omega_j}{\sqrt{|\chi_j|}}r\right\}e^{-i\sqrt{|\chi_j|}\,r}. \qquad (39)$$

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Thus we observe in the channels space-pulsations or beats with the cycle

$$\Delta r = \frac{\pi}{\Omega_j}\sqrt{|\chi_j|}. \qquad (40)$$

The correspondence principle (20) permits drawing a diagram of the pulsations between rays, if the above-mentioned normal waves refer to the central wave of the two wave packets. The widths of the beams in Figure 2 are proportional to their intensities. It must be noted that a complete transfer of energy takes place only when the channels are absolutely identical.

D. Escape of Energy Through Channel Walls. Zone of Silence (Skip-zone). Disappearance of Tropospheric Channels During a Drop in Frequency

Since a real barrier of a tropospheric or ionospheric channel always has a definite depth and width, real channels, of course, differ from the ideal cases under Section 2.

Figure 3 shows a surface channel between an ideal ground and ionosphere layer with a terminal electronic concentration described by means of a potential function. The part of the spectrum of proper values χ_j between W_{max} and W_{min} has a quasi-discrete character. The rest of the spectrum is uniform. The proper values of the central lines $\chi_1, \chi_2, \cdots \chi_j$ correspond to discrete lines of the noncompensated problem with an ideal barrier. We shall limit discussion to the case where $W_{max} > 0$, when there are proper negative values of χ. The form of the spectrum line is determined by the expression

$$\frac{d\bar{\chi}_j}{\pi X\left(1+\frac{\bar{\chi}_j^2}{X^2}\right)}, \qquad (41)$$

where $\bar{\chi}_j = |\chi_j| - |\chi|$ — is the distance of the given χ from the central line χ_j, and

$$X = \frac{1}{2I_1}e^{-2I_2},$$

where (42)

$$I_1 = \int_{z_1}^{z_2}\frac{dz}{\sqrt{k^2\varepsilon - |\chi_j|}}; \quad I_2 = 2\int_{z_2}^{z_3}\sqrt{k^2\varepsilon - |\chi_j|}\,dz$$

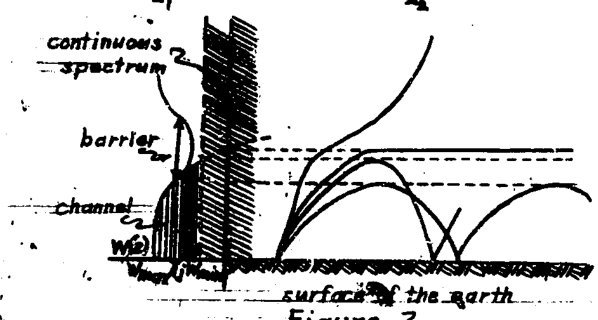

Figure 3

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Trying to excite one of the lines χ_j, corresponding to the ideal barrier, and assigning a distribution of flow $I = \Psi_j(z)$ along the fine antenna, we shall excite the whole packet of normal waves included in the quasi-discrete line of the real barrier:

$$A(r,z) \cong \frac{\psi_j(z)}{\pi \chi_j} \int_{-\Delta\chi}^{+\Delta\chi} \frac{H_0^{(2)}(\sqrt{-\chi}\,r)\, d\chi_j}{\left(1 + \frac{\bar{\chi}_j^2}{\chi^2}\right)} \tag{43}$$

After the usual simplifications permissible for great distances from the source, we shall obtain

$$A(r,z) \cong const \frac{1}{\sqrt{r}} \sum_{j=1}^{N} \frac{c_j \psi_j(z)}{\chi_j} e^{-\frac{\chi_j}{2\sqrt{|\chi_j|}} r} e^{-i\sqrt{|\chi_j|}\, r}. \tag{44}$$

Thus the effect of the escape of wave energy through the margins of the channels may be calculated by introducing the damping of the normal waves of discrete spectrum formed by function $\overline{W}(z)$.

This expression may be generalized for the case of a dissipating medium and nonconservative boundary conditions on the ground surface by introducing an additional exponential factor according to Rytov's method [6].

Treating the problem according to rays (geometric optics), we find the following expression for the damping of wave amplitude in a ray

$$\frac{1}{\sqrt{r}} e^{-(\chi_j/2\sqrt{\chi_j})r} \cdot e^{-i\psi_j}, \tag{45}$$

where r is the period of oscillation of the ray.

The continuous part of the spectrum of χ propagates rays which do not return to the ground surface. A zone of silence (skip zone) develops in a certain interval $r_1 < r < r_2$ where the effect of the continuous part of the spectrum of χ, which is responsible for the nearer antenna field, has already practically come to an end, while the quasi-discrete part of the spectrum has not yet reached the angle, since the period of oscillation r has not yet ended.

If Figure 3 is taken to refer to a tropospheric channel, characterized by the fact that the potential function W(z) does not essentially vary with frequency, it is clear that with a decrease in frequency the zero and the potential barrier for normal waves decreases. Hence, tropospheric channels vanish with decrease in frequency.

II. A SPHERICALLY STRATIFIED MEDIUM

A. Statement of the Problem -- Normal Wave Method

Let us introduce spherical coordinates R, θ, φ and observe the propagation of waves excited by radially directed currents with current-density I (R, θ) in an ideal medium, the dielectric constant of which depends only on radius R. The electromagnetic field will be determined through the scalar function A by means of equation

$$ik\varepsilon\left\{\frac{\partial}{\partial R}\left(\frac{1}{\varepsilon}\frac{\partial A}{\partial R}\right) + k^2 A\right\} + \frac{ik}{R^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial A}{\partial\theta}\right) = -\frac{4\pi}{c} I(R,\theta), \tag{46}$$

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where the electric and magnetic vectors will be defined thus:

$$H\varphi=-\frac{ik}{R}\frac{\partial A}{\partial\theta};\ E_{\theta}=\frac{1}{\varepsilon R}\frac{\partial^2 A}{\partial R\partial\theta};\ E_r=\frac{\partial}{\partial R}\left(\frac{1}{\varepsilon}\frac{\partial A}{\partial R}\right)+k^2A. \qquad (47)$$

Equation (46) may be solved in the form

$$A(R,\theta)=\sum_{j=1}^{\infty}\varphi_j(\theta)Z_j(R)+\int\varphi(\theta,\bar{\chi})Z(R,\bar{\chi})d\bar{\chi}, \qquad (48)$$

where Z_j and $Z(\bar{\chi})$ are the proper functions of the discrete and continuous part of a spectrum of proper values $\bar{\chi}$, determined by equation

$$\varepsilon R^2\left\{\frac{d}{dR}\left(\frac{1}{\varepsilon}\frac{dZ}{dR}\right)+k^2Z\right\}+\bar{\chi}Z=0, \qquad (49)$$

and $\varphi_j(\theta)$ and $\varphi(\theta,\bar{\chi})$ are Fourier coefficients satisfying equation

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\varphi_j}{d\theta}\right)-\bar{\chi}\varphi_j=-\frac{4\pi}{c}\bar{I}_j(\theta), \qquad (50)$$

where

$$\bar{I}_j(\theta)=\int I(R,\theta)Z_j(R)dR. \qquad (51)$$

The solution of (50) may be found by means of Green's function

$$K(\theta,\theta'),$$

which satisfies: (1) the boundary conditions of emission at the equator $\theta=\frac{\pi}{2}$, (2) the boundary condition of discontinuity of the first derivative at the point $\theta=\theta'$; and (3) the boundary requirements at the point $\theta=0$.

The exact value of Green's K function can be obtained only in formulating the nonstationary problem from which the principle of emission is automatically obtained, by virtue of initial conditions of Cauchy, as shown by A. A. Sokolov. The conditions here specified for K determine the wave field approximately, but not particularly the superposition of an "around-the-world" (krugosvetnyy) echo on a fundamental signal.

Under these conditions Green's function has the following form:

when $\theta>\theta'$ $K_j(\theta,\theta')=-\frac{1}{d_j}P_n(\cos\theta')L_n(\cos\theta),$ (52)

when $\theta<\theta'$ $K_j(\theta,\theta')=-\frac{1}{d_j}L_n(\cos\theta')P_n(\cos\theta),$ (53)

where $d_j=\sin\theta'[P_nL_n'-P_n'L_n],\ \bar{\chi}_j=-n(n+1,..$ [sic]

Here P_n and L_n are independent solutions of Legendre's equation (50) which, when $n\gg1$ and $\varepsilon<\theta<\pi-\varepsilon$, in an asymptotic approximation take the form:

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$$P_n(\cos\theta) = \sqrt{\frac{2}{\pi n \sin\theta}}\left[\left(1-\frac{1}{4n}\right)\sin\vartheta - \frac{1}{8n}\cot g\,\theta\cos\vartheta\right],$$

$$Q_n(\cos\theta) = \sqrt{\frac{\pi}{2n\sin\theta}}\left[\left(1-\frac{1}{4n}\right)\cos\vartheta + \frac{1}{8n}\cot g\,\theta\sin\vartheta\right], \tag{54}$$

where

$$\vartheta = \left(n+\frac{1}{2}\right)\theta + \frac{\pi}{4} \tag{55}$$

and

$$L_n(\cos\theta) = Q_n - i\frac{\pi}{2}P_n = \sqrt{\frac{\pi}{2n\sin\theta}}\left[\left(1-\frac{1}{4\pi}\right)+ + \frac{1}{8n}\cot g\,\theta\right]e^{-i\vartheta} + O\left(\frac{1}{n^{5/2}}\right), \tag{56}$$

which, at the equator $\theta = \frac{\pi}{2}$, is correct up to the third term of the asymptotic expansion and has the form of a traveling wave of propagation and hence asymptotically satisfies emission conditions. Such a function is analogous to a Hankel function of the second type and may be called a Legendre-Hankel function of the second type.

In fact, the special solution of (46) will be

$$A = \sqrt{eR}\,\psi(R)\cdot L_n(\cos\theta), \tag{57}$$

At great distances from the axis $\theta = 0$, it will take the form of a propagating wave:

$$A \cong \sqrt{eR}\,\psi(R)\sqrt{\frac{\pi}{2n\sin\theta}}\,e^{-i\left[\left(n+\frac{1}{2}\right)\theta+\frac{\pi}{4}\right]}, \tag{58}$$

where $n \cong \sqrt{-\overline{\chi}}$ for $n \gg 1$.

Thus, the proper functions of equation (49) determine the normal wave forms, and their proper values $\overline{\chi}$ are the squares of wave numbers. Non-damped propagating waves correspond to negative values of $\overline{\chi}$, and nonpropagating, rapidly damped waves to positive values of $\overline{\chi}$.

In view of the relation

$$P_n\dot{Q}_n - Q_n\dot{P}_n = \frac{1}{1-x^2}, \tag{59}$$

we obtain

$$\varphi_j(\theta) = \frac{2\pi^2}{10}\int \overline{K}(\theta,\theta')\sin\theta'\,\overline{I}_j(\theta')\,d\theta', \tag{60}$$

where $\overline{K}(\theta,\vartheta') = -d_j K(\theta,\theta')$.

If all the currents are located within the limits of a cone of angle θ_0,

$$A(r,z) = \sum C_j Z_j(R) L_n(\cos\theta), \tag{61}$$

where

$$C_j = \frac{2\pi^2}{ic}\int_0^{\theta_0} P_n(\cos\theta)\sin\theta\,\bar{I}_j(\theta)\,d\theta. \qquad (62)$$

Here, as in the previous section, we do not write out the continuous part of the normal wave spectrum.

If radially directed currents are uniformly distributed within the limits of the above-mentioned cone,

$$C_j = \frac{2\pi^2}{ic}\bar{I}\int_0^{\theta_0} P_n(\cos\theta)\sin\theta\,d\theta. \qquad (63)$$

In the case of an infinitely fine radial antenna with a definite cross section

$$C_j = \frac{\pi^2}{ic}\theta_0^2\,\bar{I}_j = \frac{\pi}{icR^2} I_j, \qquad (64)$$

where I_j is a component of the whole current I flowing through the antenna cross section

$$I_j = \int I(R)Z(R)\,dR. \qquad (65)$$

In the case of an infinitely small dipole $P = I\Delta R$, located at the point $R = R_1$, $\theta = 0$

$$A(r,\theta) = \frac{\pi P}{icR^2}\sum_{j=1}^{\infty} z_j(R_1)Z_j(R)L_n(\cos\theta). \qquad (66)$$

It is easy to demonstrate that the case of a plane-stratified medium is a limiting case of a spherically stratified medium, if the solution of the latter be presented in the form of a series in a small parameter $\mu = \frac{1}{R_e}$, where R_e is the characteristic radius lying between the return points for a wave with the least proper negative values $\bar{\chi}_N$ [1].

B. <u>Wave Guide Channels in a Spherically Stratified Medium</u> (Overground Channels Dependent on Adherence Phenomenon)

1. By substituting $Z = \sqrt{\varepsilon R}\,\Psi$ and $\xi = \ln R$, we convert equation (49) into the well-known Schrödinger-type equation

$$\frac{d^2\Psi}{d\xi^2} + [\bar{\chi} + W(\xi)]\Psi = 0, \qquad (67)$$

where the potential function has the form

$$W(\xi) = k^2\varepsilon R^2 - \sqrt{\varepsilon R}\left(\frac{1}{\sqrt{\varepsilon R}}\right)_{\xi\xi} \text{ [sic]} \qquad (68)$$

Thus in the first approximation the earlier process for channels in a plane-stratified medium can be repeated, if we substitute for the potential function $k^2 \cdot \varepsilon(z)$ the equivalent function $k^2 \cdot \varepsilon(R)R^2$; that is, we may

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introduce the equivalent dielectric constant $\varepsilon' = \varepsilon(R)R^2$ instead of $\varepsilon(z)$.

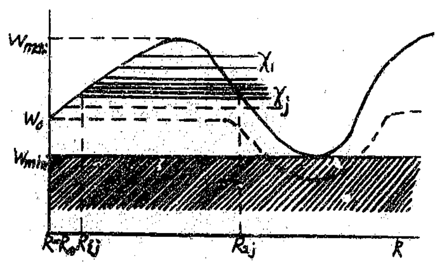

Figure 4

The dotted line in Figure 4 shows the variation, in a real case, of surface channel with height. The potential function now is

$$W(h) \cong k^2 \varepsilon R^2 \cong k^2 \cdot \varepsilon(h) \cdot (R_0 + 2h)R_0,$$

where R_0 is the ground radius, and h is the height above the ground. It may have the maximum W_{max}, if ε (h) for small values of h decreases more slowly than $\frac{1}{R_0 + 2h}$.

In this case, the first numbers with proper values in the quasi-discrete part of the spectrum propagate normal waves with amplitudes which have definite values between the two spherical surfaces $R = R_{1_j}$ and $R = R_{2_j}$. These normal waves undergo very little damping in escaping through the barrier, since the width and depth of the barrier increases with decrease of the number of the proper value. Such waves "adhere" to the concave reflecting surface forming the barrier. They predominate over other types of waves at great distances $r = R\varphi$. Waves, propagated by proper values lying in the interval $W_0 - W_{min}$ (Figure 4), have the same nature as normal waves in a plane-stratified medium.

When the potential barrier of an ionospheric layer becomes weak, which occurs at high frequencies, the quasi-discrete spectrum consists only of adherent normal waves.

2. The simplest case is to consider the ionospheric layer and the ground surface as two concentric absolutely conducting spheres with radii $R = R_0$ and $R = R_1$. If the layer between these spheres is filled with an ideal medium ($\varepsilon = 1$), equation (49) will take the form

$$\frac{d^2 Z}{dR^2} + k^2 Z + \frac{\chi}{R^2} Z = 0. \quad (69)$$

Solution (69) will be

$$Z = \sqrt{r}\,[C_1 J_p(kR) + C_2 N_p(kR)] \quad (70)$$

where J_p and N_p are Bessel and Neiman functions of the order:

$$p = \sqrt{\frac{1}{4} - \chi}. \quad (71)$$

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For greater brevity, we introduce the notation $\overline{I}_p = \sqrt{R}J_p$ and $\overline{N}_p = \sqrt{R}N_p$. Now the boundary conditions $\frac{dZ}{dR} = 0$ in the spheres $R = R_0$ and $R = R_1$ will give the transcendental equation

$$\overline{I}_p'|_{R_0} \overline{N}_p'|_{R_1} - \overline{I}_p'|_{R_1} \overline{N}_p'|_{R_0} = 0, \tag{72}$$

which determine the spectrum of proper values $\overline{\chi}_j = \frac{1-4p_j^2}{4}$. The corresponding function $Z_j(kR)$ will have the form:

$$Z_j = C_j\left[\overline{I}_{pj}(kR) - \left(\frac{\overline{I}'_{pj}}{\overline{N}'_{pj}}\right)_{R_0} \overline{N}_{pj}(kR)\right] \tag{73}$$

where C_j is determined from the condition that Z_j be normalized to the value unity.

At great distances $r = R\varphi$ we may restrict ourselves to the first N numbers of proper values in series (66), where χ_N is the smallest proper negative value. The proper values χ_j for the first numbers j propagate normal waves of the adherent type. They have approximately the form

$$Z_j \cong C_j \overline{I}_{pj}(kR), \tag{74}$$

and their proper values will be:

$$\overline{\chi}_j \cong -[kR_1 - \alpha_j \sqrt[3]{kR_1}]^2, \tag{75}$$

where $\alpha_1 = 0.8;\ \alpha_2 = 2.6;\ \alpha_3 = 3.8$.

These waves have a definite amplitude only near the outer sphere $R = R_1$. The narrow band of definite amplitudes is determined by the expression

$$\Delta\gamma_j = \beta_j \sqrt[3]{\frac{R_1}{k^2}}, \tag{76}$$

where $\beta_1 \cong 2;\ \beta_2 \cong 4;\ \beta_3 \cong 5$.

3. In the case of simple variable dielectric constant possessing the radial symmetry thus:

$$\varepsilon(R) = \frac{a^2}{R^3}. \tag{77}$$

equation (19) may be written as follows:

$$\frac{d^2Z}{dR^2} + \frac{3}{R}\frac{dZ}{dR} + \left[\frac{\overline{\chi}}{R^2} + \frac{k^2a^2}{R^3}\right]Z = 0. \tag{78}$$

The corresponding functions will be

$$Z_j = \frac{C_j}{R} J_{2\sqrt{1-\overline{\chi}_j}}\left(\frac{2ka}{\sqrt{R}}\right), \tag{79}$$

where $J_{2\sqrt{1-\overline{\chi}_j}}$ is a Bessel function of order $2\sqrt{1-\overline{\chi}_j}$. The normal wave

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spectrum in this case produces a ground-surface channel, since according to (66) definite amplitudes are observed only near the surface of the ground. The phenomenon of adherence does not take place here.

C. Radial Treatment of the Problem

The adherence phenomenon can be readily explained in terms of rays. Considering a ray as a packet of normal waves, we obtain the principle of conformity in the form

$$\frac{dS}{d\theta} = -k\sqrt{\varepsilon}\, R\sin\alpha - K\sqrt{\varepsilon(R_0)}\, R_0 \sin\alpha_0 = \sqrt{|\chi_0|}, \tag{80}$$

where α is the angle between the tangent to the trajectory of the ray and the direction of the radius R, and $\sqrt{|\chi_0|}$ is the wave number of the central normal wave of the packet forming the ray.

A packet of adherent normal waves propagates a ray which, after each full "reflection" from a stratum, re-enters the stratum without touching the surface of the ground. Its trajectory suggests a ricochet. These trajectories were first observed by Rayleigh [7] in connection with a whispering gallery. We assume that adherence phenomena play a great role in "around-the-world" echo. In addition to wave packets formed by proper values included in the interval $W_{max} > \chi > W_{min}$ (Figure 4), there are wave packets with proper values χ, included in the interval (W_0, W_{min}), which are formed by rays touching the ground. These waves are essential in studying the problem of further radio connections in a spherical-stratified medium. In this case, we can make a more accurate picture of rays, in view of the effect of the escape of wave energy through the upper "wall" of the surface channel and the discrete energy of the inner channel. Energy damping in a ray is now expressed by the formula

$$E = \frac{c}{\sqrt{\sin\theta}}\, e^{-\delta_1\theta} e^{-\delta_2\theta}. \tag{81}$$

If the medium between the receiver and transmitter is not absolutely spherically stratified and is a slowly varying function, it is possible to introduce modulated normal waves and to write the attenuation in the ray as follows.

$$E = \frac{c}{\sqrt{\sin\theta}}\, e^{-\int \delta_1 d\theta} e^{-\int \delta_2 d\theta}. \tag{82}$$

Dividing the ray trajectory between the receiver and the transmitter into parts which can each be approximated individually into a spherically stratified medium, we shall obtain by replacing integration by finite summation:

$$E = \frac{c}{\sqrt{\sin\Theta}}\, e^{-\sum_i \delta_1{}^i \Theta_i} e^{-\sum_i \delta_2{}^i \Theta_i}, \tag{83}$$

where Θ represents coefficients of the period of oscillation of the ray. The constant c can be determined by formula (63).

Formula (83) corroborates the empirical methods of calculating further connections introduced by Shchukin [8] and Eckersley [9] and points the way to improvements.

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In conclusion, it is necessary once more to emphasize that the problem of wave propagation in a spherically stratified medium has been considered as an approximation from the very beginning, since normal waves satisfy emission conditions only asymptotically for large negative proper values of χ. This does not detract from the practical value of the results, since further connections are supplied precisely by waves with large χ values.

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